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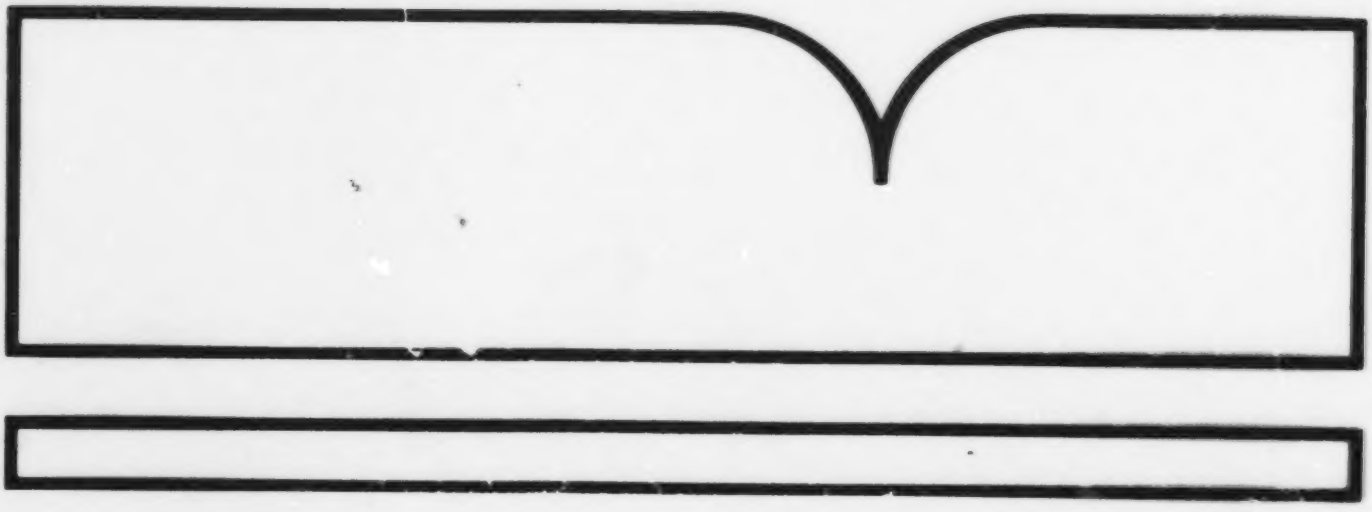

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Correction factors are calculated for obtaining true scattering extinction coefficients from integrating nephelometer measurements. The corrections are based on the bimodal representation of ambient aerosol size distributions, and take account of the effects of angular truncation and sampling efficiency cutoff. Numerical values of correction factors appropriate to the conventional governing equation of the integrating nephelometer are shown to be indistinguishable from those of the more complex governing equation of Rabinoff and Herman.

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Integrating Nephelometer Response Corrections for Bimodal Size Distributions

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Correction factors are calculated for obtaining true scattering extinction coefficients from integrating nephelometer measurements. The corrections (specific to MRI 1560/1590 instruments) are based on the bimodal representation of ambient aerosol size distributions and take into account the effects of angular truncation and sampling efficiency cutoff. Numerical values of correction factors appropriate to the conventional governing equation of the integrating nephelometer are shown to be indistinguishable from those of the more complex governing equation of Rabinoff and Herman.

INTRODUCTION

Integrating nephelometers are widely used in studies of air quality. Their usefulness arises from their capability to provide a relatively simple measurement of the scattering extinction coefficient for particles (i.e., volume scattering coefficient), a quantity of considerable importance in atmospheric aerosol characterization in general and human visibility concerns in particular. The conventional theory of the integrating nephelometer results in a direct proportionality between the response of the nephelometer and the scattering coefficient. There are two well-known effects, however, that in practice distort the theoretical proportionality:

1. In angular truncation error, the instrument's internal geometry causes light scattered near 0° and 180° to be lost from the assumed (0°–180°) full range of scattering.
2. In sampling efficiency error, large particles that contribute to extinction in the atmosphere do not contribute to the instrument's response, owing to the reduced efficiency with which these particles are transported to the instrument's sensing volume.

Both effects separately cause the measured scattering coefficient to be systematically smaller than the true value. Previous investigators (Ensor and Waggoner, 1970; Heintzenberg and Quenzel, 1973; Fitzgerald, 1977; and references therein) have estimated the size of both error types, although not in a unified way, and generally only for a power-law representation of the aerosol size distribution. Since the bimodal aerosol size distribution is now generally regarded as a better approximation for tropospheric aerosol, we have calculated a set of correction factors, incorporating both types of errors, which cover a useful range of log-normal variables in terms of which the bimodal distribution is parameterized. The correction factors are applicable to the model 1560 and 1590 series integrating nephelometers manufactured by Meteorology Research Inc.

A second objective of the present work was to investigate the observation of Rabinoff and Herman (1973) that the conventional integrating nephelometer theory contains a subtle error in its derivation, apart from the two effects already noted. A critical evaluation of the Rabinoff–Herman (RH) approach leads us to conclude that, at least within the context of the bimodal aerosol

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model, any such error causes virtually no change from the numerical results of the conventional theory.

The present work was motivated by an interest in determining the *absorption* extinction coefficient by a difference method requiring the scattering extinction coefficient to be known to high accuracy. Results from measurements performed in Denver during January 1982, using the correction factors developed in the present work, will be reported elsewhere.

THEORY

Conventional Approach

The scattering coefficient b for a collection of aerosol particles in the atmosphere may be defined by

$$b = 2\pi \int_0^{\pi} \beta(\Theta) \sin\Theta \, d\Theta, \tag{1}$$

where $\beta(\Theta)$ is the scattering function, i.e., the scattering cross section per unit volume of aerosol per unit solid angle for unpolarized light scattered through an angle Θ. The scattering function is itself an integral over the entire particle size distribution. The conventional derivation (Middleton, 1963) of the governing equation for the integrating nephelometer leads to a remarkably simple relation between b and the output B (power per unit solid angle per unit area of detector surface) of the instrument,

$$B = (I_0/2\pi r_0) b \tag{2}$$

where r_0 is the perpendicular distance from the light source to the axis of the viewing cone of the detector (Figure 1), and I_0 is the power per unit solid angle emitted by the source along the direction of r_0.

In practice, however, (2) is distorted by two effects: angular truncation error and sampling efficiency error. These lead to a *measured* output B' given by

$$B' = \frac{I_0}{r_0} \int_{\Theta_1}^{\Theta_2} \beta'(\Theta) \sin\Theta \, d\Theta, \tag{3}$$

with Θ_1 and Θ_2 being the instrument's truncation angles (Figure 1), and

$$\beta'(\Theta) = \frac{3}{4\pi} \int_{-\infty}^{\ln D_{\max}} \left(\frac{\lambda}{\pi D}\right)^2 [i_1(\Theta) + i_2(\Theta)] \times \frac{1}{D} \frac{dV}{d(\ln D)} d(\ln D), \tag{4}$$

where

λ is the wavelength of light,

D is the (geometrical) particle diameter,

$D_{\max}$ is the (geometrical) sampling efficiency cutoff diameter,

$[dV/d(\ln D)]\, d(\ln D)$ is the aerosol volume per unit volume for particles with diameters in the range between $\ln D$ and $\ln D + d\ln D$,

and $i_1(\Theta)$, $i_2(\Theta)$ are Mie intensity functions as used by Ensor and Waggoner (1970). The expression for $\beta'(\Theta)$ differs from that of $\beta(\Theta)$ in (1) only in the upper limit of the integral, which is infinite in the case of $\beta(\Theta)$. The form of (4), involving the volumetric rather than the number representation of the ambient aerosol size distribution, has been chosen in anticipation of its convenience in the calculations to follow.

The *measured* extinction coefficient b' is then taken to be

$$b' = k'B'. \tag{5}$$

The customary way in which the proportionality constant k' is determined is to employ a calibration gas whose scattering function $\beta_c(\Theta)$ is known and to require

$$b_c' = b_c, \tag{6}$$

where b_c' is the extinction coefficient measured for the calibration gas, and b_c is the true extinction coefficient for the calibration gas. Thus k' can be written

$$k' = (2\pi r_0/I_0) k, \tag{7}$$

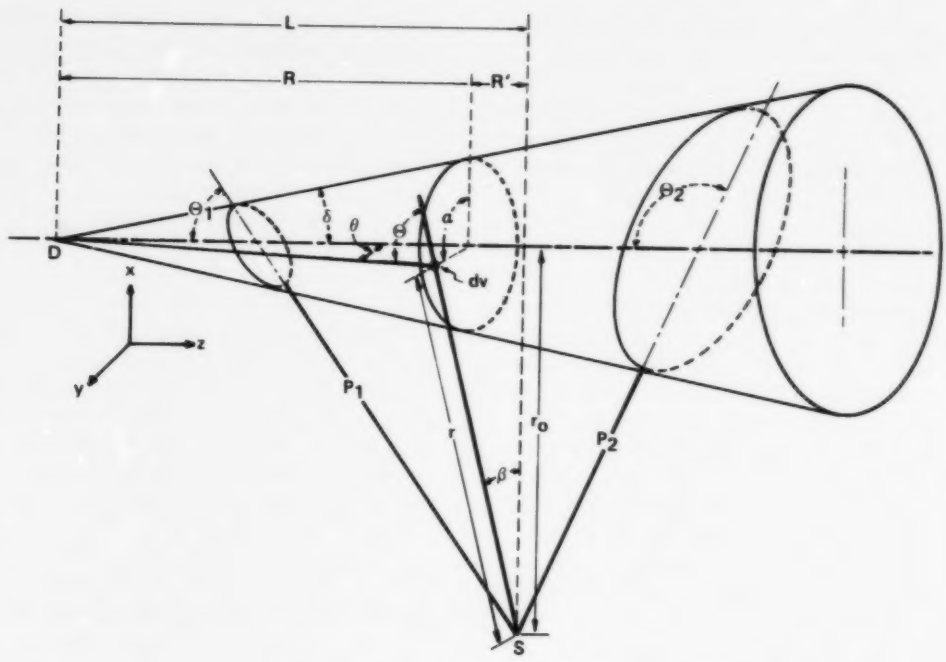

FIGURE 1. Nephelometer optical geometry.

where

$$k = \int_0^{\pi} \beta_c(\Theta)\sin\Theta\, d\Theta \times \left(\int_{\Theta_1}^{\Theta_2} \beta_c(\Theta)\sin\Theta\, d\Theta\right)^{-1}. \quad (8)$$

If we now define for the integrating nephelometer an aerosol correction factor

$$F = b'/b, \quad (9)$$

and if F is known as a numerical value, then the true extinction coefficient b_{true} corresponding to a measured value b_{meas} can be determined from the equation

$$b_{true} = b_{meas}/F. \quad (10)$$

Equation (10) is the fundamental relationship in this study. The quantity F depends on the *relative* size distribution of the aerosol particles in the atmosphere, their index of refraction, and the integrating nephelometer parameters λ, Θ_1, Θ_2, and D_{max}. More explicitly, F can be written

$$F = kf \quad (11)$$

with k given by (8) and f defined as

$$f = 2\pi \int_{\Theta_1}^{\Theta_2} \beta'(\Theta)\sin\Theta\, d\Theta \times \left[\frac{3}{2}\int_{-\infty}^{+\infty} Q_s \frac{1}{D}\frac{dV}{d(\ln D)}\, d(\ln D)\right]^{-1}. \quad (12)$$

In (12) Q_s is known as the scattering efficiency factor. The denominator of (12) is an alternative definition of the true scattering coefficient b. The advantage of this definition over (1) is that a single rather than double integration is needed to evaluate b. Equation (12) would have a more symmetric appearance if (1) were used to represent b, but the resulting expression would require more effort in its calculation. Note that (11) has a

convenient structure with all the calibration influence residing in a separate factor.

Except for our additional inclusion of the sampling efficiency cutoff effect, the manner in which the correction factor F was obtained is essentially the same as the approach presented by Ensor and Waggoner (1970). Those authors assumed a power-law representation of the ambient aerosol size distribution and calculated a set of correction factors corresponding to several choices for the exponent. It is now generally accepted that a better representation of ambient aerosol size distributions in the troposphere is a bimodal one, consisting of a sum of more or less independent amounts of a fine-particle and a coarse-particle mode. If the size distribution is written in a volumetric representation and the components are represented by log-normal functions, then the *relative* size distribution for the bimodal case can be written

$$\frac{dV}{d(\ln D)} = \frac{dV_1}{d(\ln D)} + R\frac{dV_2}{d(\ln D)}, \qquad (13)$$

where

$$\frac{dV_i}{d(\ln D)} = \frac{1}{(2\pi)^{1/2}\ln\sigma_i} \times \exp\left[-\left(\frac{\ln D}{\mathrm{DGV}_i}\right)^2 \Big/ 2\ln^2\sigma_i\right], \qquad (14)$$

where $i = 1$ for fine mode, $i = 2$ for coarse mode, and R is the ratio of the total volume (not mass) of coarse to fine aerosol.

Rabinoff–Herman Approach

Nearly a decade ago Rabinoff and Herman (1973) presented a new derivation of the governing equation for the integrating nephelometer, obtaining an expression very different in appearance from the result (2) of the conventional derivation. The RH approach aimed to avoid a questionable assumption required in the conventional derivation, that the scattering angle is constant at every point within each differential scattering volume formed by the viewing cone of the detector and two infinitesimally separated planes perpendicular to the cone's axis. This assumption is only true in the limit as the cone angle of view of the detector vanishes. While the RH theory is not unknown to workers in this field (Fitzgerald, 1977; Ruby and Waggoner, 1981) its complexity seems to have discouraged both its application or any attempt to critically evaluate its correctness. Since our overall objective was to develop the best possible set of correction factors for deriving true scattering coefficients from integrating nephelometer measurements, it was natural to undertake such an evaluation. In doing so we have corrected an error that is apparent in the RH presentation, and have modified the derivation to include the effect of sampling efficiency cutoff, in addition to the angular truncation effect already implicitly contained in their original derivation.

The RH derivation is based on the representation of the integrating nephelometer's internal geometry as given in Figure 1. A point source of light is at S, and a detector at D views a volume ΔV defined by the cone and the two planes P_1 and P_2. Planes P_1 and P_2 simulate the influence of a system of baffles in a real nephelometer which limit the viewing volume. For ease of comparison the symbols shown in Figure 1 (as well as subsequent variables defined in terms of them) are identical with those chosen by Rabinoff and Herman. (Note that the symbol β is used for two different quantities and that Θ must be distinguished from θ.)

The RH approach leads to an expression for the *measured* nephelometer response of the form

$$B'_{\mathrm{RH}} = \frac{I_0}{\omega_{\mathrm{cone}}}\int_{\Delta V}\frac{\cos\beta\cos\theta\beta'(\Theta)}{r^2R_{\mathrm{d}}^2}dV, \qquad (15)$$

where I_0 and $\beta'(\Theta)$ have the same meanings as before, ω_{cone} is the solid angle of the cone,

$$\omega_{cone} = 2\pi(1 - \cos\delta), \tag{16}$$

and

$$R_d = R\sec\theta. \tag{17}$$

Expressions are given by Rabinoff and Herman for the quantities dV, $\cos\Theta$, r, and $\cos\beta$ in terms of the basic coordinates R, θ, and α. In a subsequent corrigendum, Rabinoff and Herman (1974) give different expressions for $\cos\Theta$ and r. The revised result for $\cos\Theta$ is still incorrect, however, and should be replaced by

$$\cos\Theta = \frac{R' - R\tan^2\theta - r_0\tan\theta\cos\alpha}{r\sec\theta}. \tag{18}$$

The two differ in the algebraic sign of R'; the correctness of (18), however, may be easily seen by considering Figure 1 for the special case in which $\theta = 0$. Finally, we note that the correct expression for R' is

$$R' = L - R. \tag{19}$$

Equation (15) differs from the RH result only in the replacement of $\beta(\Theta)$ by $\beta'(\Theta)$ as given by (4). Thus (15) includes not only the distorting effect of angular truncation, implicitly contained in the original RH derivation, but the effect of sampling efficiency cutoff as well. Consequently Eqs. (15) and (3), while very different in appearance, describe exactly the same quantity, including the same distorting effects, but from the different viewpoints of the RH approach and the conventional theory of the integrating nephelometer.

We may then proceed in steps parallel to those following (5) to obtain the correction factor F_{RH} appropriate to the RH approach:

$$F_{RH} = k_{RH}f_{RH}, \tag{20}$$

where

$$k_{RH} = \int_0^{\pi}\beta_c(\Theta)\sin\Theta\,d\Theta \times\left[\frac{r_0}{\omega_{cone}}\int_{\Delta V}\frac{\cos\beta\cos\theta\beta_c(\Theta)\,dV}{r^2R_d^2}\right]^{-1} \tag{21}$$

and

$$f_{RH} = \frac{2\pi r_0}{\omega_{cone}}\int_{\Delta V}\frac{\cos\beta\cos\theta\beta'(\Theta)\,dV}{r^2R_d^2} \times\left[\frac{3}{2}\int_{-\infty}^{+\infty}Q_s\frac{1}{D}\frac{dV}{d(\ln D)}d(\ln D)\right]^{-1}. \tag{22}$$

With F_{RH} known, it can be used in place of F in (10) to obtain the true scattering coefficient from the measured one. Once again the details of the specific calibration procedure are collected in a separate multiplicative quantity (k_{RH}). This quantity is absent from the correction factor developed by Rabinoff and Herman, which accounts for the otherwise puzzling observation that their correction factor does not approach unity for the case of a Rayleigh scatterer.

There is, of course, no reason to expect the two versions of the correction factor, F and F_{RH}, to be identical since each is based on a somewhat different mathematical model of a real integrating nephelometer. The advantage of F_{RH} is that its basis is mathematically more rigorous, while the advantage of F is that it is simpler to calculate. It remains, then, to calculate F and F_{RH} for a series of environmentally interesting cases and to compare the results.

RESULTS

Except as otherwise noted, the numerical results that follow refer to the model 1590 series of integrating nephelometers (Meteorology Research Inc.). The results should apply to the 1560 series as well, since they

TABLE 1. Physical Constants of the MRI 1560/1590 Integrating Nephelometer

λ	0.53 μm
Θ_1	8°
Θ_2	168°
δ	0.47°
r_0	32 mm
L	525 mm

TABLE 2. Comparison of $1 - F$ and T_{AC}

	$1 - F$		
n	step[a] = 0.069	step[a] = 0.0138	T_{AC}[b]
2	0.2525	0.2537	0.21
3	0.0833	0.0841	0.091
4	0.0286	0.0289	0.028

[a] Integration step size, $\Delta \ln D$.
[b] From Ensor and Waggoner (1970, Table 1, column 4).

are optically identical instruments. Values for the relevant parameters of the instrument needed in the calculations are given in Table 1. They were obtained by direct measurement on a 1591 and a 1597 unit.

CALCULATION OF F

Evaluation of k

Evaluation of the factor k occurring in the expression for F requires specification of the calibrating gas scattering function $\beta_c(\Theta)$. Because freon-12 continues to be the most commonly used gas for this purpose we have taken $\beta_c(\Theta)$ appropriate to this gas. This gives (McCartney, 1976)

$$[\beta_c(\Theta)]_{\text{freon-12}} \propto 1 + 0.9084 \cos^2 \Theta, \qquad (23)$$

where the coefficient of $\cos^2 \Theta$ is derived from the depolarization factor for freon-12 (0.048) measured by Bodhaine (1979). Using (23), we obtain from (8)

$$[k]_{\text{freon-12}} = 1.023. \qquad (24)$$

This value is within 0.1% of that for a pure Rayleigh scattering gas ($\cos^2 \Theta$ coefficient of unity),

$$[k]_{\text{Rayleigh}} = 1.024. \qquad (25)$$

Evaluation of f

Power-Law Size Distribution. The remainder of the calculation of F was done by numerical integration of the two integrals appearing in (12) using Simpson's rule and employing an existing FORTRAN subroutine DBMIE (Dave, 1968) for generation of the functions i_1, i_2, and Q_s. As a check of the correctness of the overall computer program used, we first attempted to reproduce some of the correction factors calculated by Ensor and Waggoner (1970) for a power-law aerosol size distribution ($dV/d\log D \propto D^{-n+3}$). The comparison is shown in Table 2, where their correction factor T_{AC} and our F should be related according to

$$T_{AC} = 1 - F. \qquad (26)$$

The parameters (integration limits, refractive index, wavelength, and nephelometer constants) used in the calculation of F were the same as those used in the earlier work, similar but not identical to those of Table 1. The step size for the Θ integration was 0.5°. The adequacy of this choice was established by noting excellent agreement between Q_s calculated directly by DBMIE compared with a numerical integration of $i_1(\Theta) + i_2(\Theta)$ over all angles. Table 2 gives results for two choices of step size, differing by a factor of 5, over the variable $\ln D$. The results for the two choices agree very well with each other and somewhat less well with the previous calculations given in the last column of Table 2. Most of the latter difference may be due to compromises in the calculations of T_{AC} that were dictated by reasons of economy (Ensor, 1982). The earlier results were certainly adequate, however, in the context of their time.

Bimodal Size Distribution. The correction factors F appropriate to bimodal size distributions were obtained by using (13) in conjunction with (4), (11), (12), and (24). In

TABLE 3. Correction Factors F for MRI 1560/1590 Integrating Nephelometer[a]

DGV_1 \ DGV_2	$D_{max} = 1$		$D_{max} = 2$		$D_{max} = 5$		$D_{max} = 15$		
	6	12	6	12	6	12	6	12	R
0.2	0.96	0.97	0.97	0.97	0.97	0.98	0.97	0.98	0.2
	0.87	0.93	0.89	0.94	0.92	0.94	0.92	0.95	1
	0.79	0.88	0.82	0.89	0.86	0.90	0.87	0.91	2
	0.72	0.84	0.76	0.85	0.82	0.86	0.83	0.88	3
	0.62	0.77	0.67	0.78	0.75	0.80	0.77	0.82	5
	0.46	0.63	0.54	0.64	0.64	0.68	0.67	0.71	10
0.3	0.94	0.95	0.96	0.97	0.96	0.97	0.97	0.97	0.2
	0.88	0.92	0.91	0.94	0.92	0.94	0.93	0.95	1
	0.82	0.89	0.85	0.90	0.88	0.91	0.89	0.92	2
	0.76	0.86	0.80	0.87	0.85	0.89	0.86	0.89	3
	0.67	0.80	0.73	0.82	0.79	0.84	0.80	0.85	5
	0.53	0.69	0.60	0.71	0.69	0.74	0.71	0.76	10
0.5	0.90	0.91	0.94	0.95	0.95	0.95	0.95	0.95	0.2
	0.85	0.88	0.90	0.92	0.91	0.93	0.92	0.93	1
	0.80	0.86	0.85	0.89	0.88	0.90	0.88	0.91	2
	0.75	0.83	0.81	0.87	0.84	0.88	0.85	0.89	3
	0.67	0.78	0.74	0.82	0.79	0.84	0.81	0.85	5
	0.53	0.68	0.62	0.72	0.70	0.75	0.72	0.77	10

[a] $\lambda = 0.530\ \mu m$, $\sigma_1 = \sigma_2 = 2$. D_{max}, DGV_1, and DGV_2 are in microns.

choosing an appropriate set of parameters for the general bimodal distribution expression (13) our objective was to be generally consistent with the grand average continental values given by Whitby (1978) and in addition to consider an environmentally plausible range of values for at least some of the parameters. The resulting values for F are given in Table 3. The values pertain to the 1560/1590 integrating nephelometer constants of Table 1 and assume an aerosol refractive index of $1.5 - 0.02i$. Step sizes for the Θ and $\ln D$ numerical integrations were 0.5° and approximately 0.04, respectively. The infinite limits for the integrals in (4) and (12) were approximated by the logarithm corresponding to D values of 0.01 and 100 μm.

The largest value for D_{max} specified in Table 3 is 15 μm. Calculations were also performed for 50 μm, but the resulting F values were found to be virtually identical to those for 15 μm. Thus, extending the sampling efficiency cutoff of an integrating nephelometer beyond 15 μm will not improve the accuracy of the measured scattering extinction coefficient.

Figure 2 displays in graphical form a subset of the entries in Table 3 chosen to illustrate some of the trends in the correction factor. As expected, F is always smaller than unity, it decreases with increasing loading of the coarse particle mode, and it increases as the sampling efficiency cutoff increases. Because of the asymptotic character of the 15 μm curve, it is easy to determine the relative importance of the two effects that contribute to the correction factor. For example, in the case of a 1-μm cutoff and equal volumetric loadings of fine and coarse modes ($R = 1$), the total correction factor is 12% smaller than unity, of which 7% is contributed by angular truncation and 5% is contributed by sampling efficiency cutoff.

Use of Table 3 (or Figure 2) requires an estimate of D_{max}. With a well-designed sampling head (e.g., Liu and Pui, 1981) and vertical ducting to the nephelometer inlet, D_{max} of at least 10 μm is achievable. Even in the case of a more tortuous sampling line,

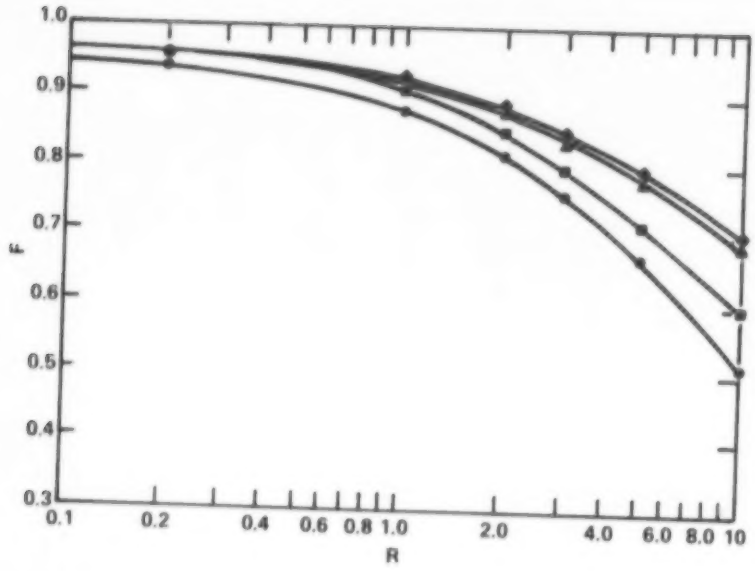

FIGURE 2. Correction factor F versus ratio R of volume concentration of coarse to fine aerosol for effective sampling cutoffs of 1 (●), 2 (■), 5 (▲), and 15 (◆) μm. $\lambda = 0.53$ μm; $DGV_1 = 0.3$ μm and $DGV_2 = 6$ μm; $\sigma_1 = \sigma_2 = 2$; $m = 1.5 - 0.02i$.

the nominal flow rate (140 l min^{-1}) and inlet diameter (4 cm) of MRI 1560/1590 integrating nephelometers lead to a surprisingly efficient transfer of large particles, as can be inferred from the theoretical calulations of Crane and Evans (1977) on the transmission of aerosol through a 90° bend in a pipe of circular cross section. Thus in the typical situation D_{max} will be determined by the sampling head, and a value in the range 2–5 μm is likely if no particular effort is made in the head's design.

It has proven useful in previous measurements with integrating nephelometers to operate the instrument at more than one wavelength (Ensor et al., 1972). Tables 4 and 5 give correction factors appropriate to two additional wavelengths, 0.675 and 0.436 μm, respectively. All other geometrical and aerosol parameters used in the calculations were identical with those of Table 3.

CALCULATION OF F_{RH}

To perform the integrations over the complicated three-dimensional volume required in the calculation of F_{RH}, a Monte Carlo method was used. In the general case a Monte Carlo approximation to the integral

$$A = \int_{\Delta V} I(x, y, z)\, dx\, dy\, dz \tag{27}$$

is

$$A \cong \frac{\Delta V}{N} \sum_{i=1}^{N} I(x_i, y_i, z_i), \tag{28}$$

where each three-dimensional point (x_i, y_i, z_i) must be chosen at random from within ΔV. Thus, in general this method requires a calculation of the quantity ΔV as a preliminary step to obtaining A and is typically accomplished by an additional Monte Carlo calculation. In the particular problem at hand the intermediate calculation of ΔV could be avoided since ΔV occurs in the Monte Carlo approximations to both the denominator of k_{RH} and the numerator of f_{RH}, and thus would cancel in the product (20). We chose, however, not to take advantage of this simplification for two reasons: First, in this case

TABLE 4. Correction Factors F for $\lambda = 0.675\ \mu m^a$

DGV_1 \ DGV_2	$D_{max} = 1$		$D_{max} = 2$		$D_{max} = 5$		$D_{max} = 15$		
	6	12	6	12	6	12	6	12	R
0.2	0.94	0.96	0.96	0.97	0.97	0.97	0.97	0.98	0.2
	0.80	0.89	0.84	0.90	0.89	0.92	0.90	0.93	1
	0.68	0.82	0.74	0.83	0.82	0.86	0.84	0.87	2
	0.60	0.75	0.67	0.77	0.77	0.80	0.79	0.83	3
	0.48	0.66	0.57	0.68	0.70	0.72	0.73	0.76	5
	0.33	0.49	0.45	0.52	0.62	0.58	0.66	0.64	10
0.3	0.92	0.94	0.96	0.97	0.97	0.97	0.97	0.97	0.2
	0.83	0.89	0.88	0.92	0.91	0.93	0.92	0.94	1
	0.74	0.84	0.80	0.87	0.86	0.89	0.87	0.90	2
	0.67	0.80	0.74	0.83	0.82	0.85	0.84	0.87	3
	0.56	0.72	0.65	0.75	0.76	0.79	0.78	0.81	5
	0.41	0.58	0.52	0.61	0.67	0.67	0.70	0.71	10
0.5	0.86	0.87	0.95	0.95	0.95	0.96	0.96	0.96	0.2
	0.80	0.84	0.89	0.92	0.91	0.93	0.92	0.94	1
	0.73	0.81	0.82	0.88	0.87	0.90	0.88	0.91	2
	0.67	0.77	0.77	0.85	0.84	0.87	0.85	0.88	3
	0.58	0.71	0.69	0.79	0.78	0.81	0.80	0.83	5
	0.44	0.60	0.57	0.66	0.70	0.71	0.73	0.75	10

[a] $\sigma_1 = \sigma_2 = 2$. D_{max}, DGV_1, and DGV_2 are in microns.

TABLE 5. Correction Factors F for $\lambda = 0.436\ \mu m^a$

DGV_1 \ DGV_2	$D_{max} = 1$		$D_{max} = 2$		$D_{max} = 5$		$D_{max} = 15$		
	6	12	6	12	6	12	6	12	R
0.2	0.96	0.97	0.97	0.97	0.97	0.97	0.97	0.98	0.2
	0.91	0.94	0.92	0.95	0.93	0.95	0.93	0.95	1
	0.84	0.91	0.86	0.91	0.89	0.92	0.89	0.93	2
	0.79	0.88	0.82	0.89	0.85	0.89	0.86	0.90	3
	0.70	0.83	0.74	0.83	0.79	0.84	0.80	0.86	5
	0.55	0.71	0.61	0.72	0.68	0.75	0.70	0.77	10
0.3	0.95	0.95	0.96	0.96	0.96	0.96	0.96	0.96	0.2
	0.90	0.93	0.92	0.94	0.93	0.94	0.93	0.95	1
	0.85	0.91	0.88	0.92	0.89	0.92	0.90	0.93	2
	0.81	0.88	0.84	0.89	0.86	0.90	0.87	0.91	3
	0.73	0.84	0.77	0.85	0.81	0.86	0.82	0.87	5
	0.60	0.74	0.65	0.76	0.71	0.78	0.73	0.79	10
0.5	0.91	0.91	0.94	0.94	0.94	0.94	0.94	0.94	0.2
	0.86	0.89	0.90	0.92	0.91	0.92	0.91	0.93	1
	0.82	0.87	0.86	0.90	0.88	0.90	0.88	0.91	2
	0.78	0.85	0.82	0.87	0.85	0.88	0.85	0.89	3
	0.71	0.80	0.76	0.83	0.79	0.84	0.81	0.85	5
	0.58	0.72	0.64	0.74	0.70	0.76	0.72	0.78	10

[a] $\sigma_1 = \sigma_2 = 2$. D_{max}, DGV_1, and DGV_2 are in microns.

ΔV can be calculated analytically and then compared with the numerical Monte Carlo result to provide a check on the overall correctness of the Monte Carlo computer program, and second, it is of interest to calculate k_{RH} explicitly so that it may be directly compared with k, providing one more piece of information on the general relationship between the conventional and RH nephelometer theories.

Evaluation of ΔV

Analytical Result. Relative to the cartesian coordinate system whose origin is at D (Figure 1), the volume ΔV includes those points (x, y, z) that simultaneously satisfy the inequalities

$$z^2/(x^2+y^2) \geq a^2, \tag{29}$$

$$z + b_1 x \geq C_1, \tag{30}$$

$$z + b_2 x \leq C_2, \tag{31}$$

where

$$a = \cot \delta, \tag{32}$$

$$b_i = \cot \Theta_i, \tag{33}$$

$$C_i = L - r_0 \cot \Theta_i \tag{34}$$

with $i = 1$ or 2.

Although we shall not give the details here, the theory of conic sections leads to the result

$$\Delta V = \frac{\pi a}{3}\left[\frac{C_2^3}{(a^2-b_2^2)^{3/2}} - \frac{C_1^3}{(a^2-b_1^2)^{3/2}}\right]. \tag{35}$$

With reference to the numerical values of Table 1, we have

$$\Delta V = 19{,}913 \quad \text{mm}^3. \tag{36}$$

Monte Carlo Result. The first step in a Monte Carlo calculation of ΔV was to circumscribe a rectangular parallelopiped of volume V_R about the cone shown in Figure 1. If a succession of N points are chosen randomly from within the parallelopiped, then ΔV is approximately given by

$$\Delta V \cong N_{in} V_R / N, \tag{37}$$

where N_{in} is the number of randomly chosen points that also lie within ΔV, i.e., satisfy inequalities (29)–(31). Equation (37) is, of course, a special case of (28) for which $I(x_i, y_i, z_i)$ is either zero or unity. The points were chosen with the aid of the Univac-supported random number generator subroutine RANDU. For $N = 10^4$ the result was

$$\Delta V = (19{,}914 \pm 27) \quad \text{mm}^3, \tag{38}$$

where the value is the mean of ten independent calculations with $N = 10^4$ for each, and the error is the standard deviation of the mean. The result (38) is in excellent agreement with (36).

Evaluation of k_{RH}

To apply the Monte Carlo expression (28) to the evaluation of the denominator of (21), the same volume V_R used previously was employed. Successive points from within V_R were chosen by RANDU until one was found that was also internal to ΔV, i.e., that satisfied inequalities (29)–(31). The integrand was evaluated at that point and added to the sum (28). This sequence was repeated 10^4 times ($N = 10^4$), and the resulting sum was combined with the value for ΔV from (36) to complete the integral evaluation. The result for k_{RH} was

$$[k_{RH}]_{\text{fregon-12}} = 1.024 \pm 0.001, \tag{39}$$

where the value is the mean of ten independent evaluations, and the error is the standard deviation of the mean. The result for k_{RH} is thus numerically indistinguishable from the corresponding result, (24), from the conventional theory.

Evaluation of f_{RH}

The Monte Carlo calculation of the numerator in (22) was performed in a manner simi-

lar to that just described, except for the treatment of $\beta'(\Theta)$. For each set of parameters, including D_{max} and the log-normal constants chosen for the aerosol size distribution, $\beta'(\Theta)$ was precalculated from (4) in steps of 0.1°, using Simpson's rule with an integration step size of approximately 0.04 in ln D. Then in the Monte Carlo procedure, for each particular Θ_i at which it was required to evaluate the integral, $\beta'(\Theta_i)$ was determined by interpolation.

Comparison of F_{RH} with F

Values of f_{RH}, and subsequently F_{RH}, were computed for a total of 12 cases, corresponding to the $DGV_1 = 0.3$ μm portion of columns 1 and 7 in Table 3. Nine of the F_{RH} values were identical with the corresponding entries in Table 3, and the remaining three differed by at most 0.01. There was little incentive to proceed with further comparison, since it appeared likely that no significant differences would be found in the correction factors calculated from the two different approaches.

SUMMARY

The main results of this work are the following. First, a set of correction factors (Table 3) for obtaining true scattering extinction coefficients from MRI model 1560/1590 integrating nephelometer measurements have been calculated. The correction factors are appropriate to bimodal aerosol size distributions and include the distorting effects of both angular truncation error and sampling efficiency cutoff error. Second, an alternative governing equation for the integrating nephelometer, as proposed by Rabinoff and Herman, has been critically evaluated and shown to lead to correction factors that are numerically indistinguishable from those of the conventional governing equation.

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